1. Dada la matriz

$$A=\begin{pmatrix}2&2&1\\1&3&1\\1&2&2\end{pmatrix}$$

obtenga sus autovalores y autovectores. Razone si es diagonalizable y, en caso afirmativo, escriba las matrices D y P que cumplen que $D=P^{-1}AP$.

$\det(A-\lambda I)=0$

Autovalor

$$\det(A-\lambda I_3)=\det\begin{pmatrix}2-\lambda&2&1\\1&3-\lambda&1\\1&2&2-\lambda\end{pmatrix}=(2-\lambda)\cdot(3-\lambda)\cdot(2-\lambda)+2+2-((3-\lambda)-4-2\lambda-4-2\lambda)=\boxed{-\lambda^3+7\lambda^2-11\lambda+5}$$

$$\begin{array}{r|rrrr} & -1 & 7 & -11 & 5\\ 1 & & -1 & 6 & -5\\ \hline & -1 & 6 & -5 & 0\end{array} \qquad \boxed{\lambda=1}$$

$-\lambda^2+6\lambda-5=0$; $\lambda=\dfrac{-6\pm\sqrt{36-20}}{-2}=\begin{cases}\lambda=5\\ \lambda=1\end{cases}$

$\boxed{\begin{array}{l}\lambda=1\\ \lambda=1\\ \lambda=5\end{array}}$

2 repetidos, así que $\lambda=1$ tendremos que sacar 2 autovectores.

Autovector

$\lambda=1 \longrightarrow A-I_3=\begin{pmatrix}1&2&1\\1&2&1\\1&2&1\end{pmatrix}$; $\left.\begin{array}{l}x+2y+z=0\\ \cancel{x+2y+z=0}\\ \cancel{x+2y+z=0}\end{array}\right\}$ $\vec{V_1}=(1,0,-1)$ • $\vec{V_2}=(2,-1,0)$ •

$\lambda=5 \longrightarrow A-5I_3=\begin{pmatrix}-3&2&1\\1&-2&1\\1&2&-3\end{pmatrix}$; $\left.\begin{array}{l}-3x+2y+z=0\\ x-2y+z=0\\ \cancel{x+2y-3z=0}\end{array}\right\}\longrightarrow \vec{V_3}=(1,1,1)$ •

$\hookrightarrow rg(A-5I_3)=2$

Diagonalizar

$A=\begin{pmatrix}2&2&1\\1&3&1\\1&2&2\end{pmatrix}$ $\quad B=\{(1,0,-1),(2,-1,0),(1,1,1)\}$ de $\mathbb{R}^3$

$1\cdot(1,0,-1)=1\cdot(1,0,-1)+0\cdot(2,-1,0)+0\cdot(1,1,1)$

$1\cdot(2,-1,0)=0\cdot(1,0,-1)+1\cdot(2,-1,0)+0\cdot(1,1,1)$

$5\cdot(1,1,1)=0\cdot(1,0,-1)+0\cdot(2,-1,0)+5\cdot(1,1,1)$

$\Rightarrow D=\begin{pmatrix}1&0&0\\0&1&0\\0&0&5\end{pmatrix}$

$P=\begin{pmatrix}1&2&1\\0&-1&1\\-1&0&1\end{pmatrix}\longrightarrow$ Formada por autovectores en columnas.

2. Sea $f : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ la aplicación lineal definida como

$$f((x,y,z)) = (2x + y - z, 2y - z, -3x - 2y + 3z).$$

Obtenga sus autovalores y autovectores. Razone si es diagonalizable.

Para la $B_c = \{(1,0,0), (0,1,0), (0,0,1)\} \longrightarrow A = \begin{pmatrix} 2 & 1 & -1 \\ 0 & 2 & -1 \\ -3 & -2 & 3 \end{pmatrix} \rightarrow$ Aplicar f a vects. de la B_c

$\boxed{\text{Autovalores}}$

$$\det(A - \lambda I_3) = \begin{pmatrix} 2-\lambda & 1 & -1 \\ 0 & 2-\lambda & -1 \\ -3 & -2 & 3-\lambda \end{pmatrix} = (2-\lambda)^2 \cdot (3-\lambda) + 3 - 6 + 3\lambda - 4 + 2\lambda = -\lambda^3 + 7\lambda^2 - 11\lambda + 5 = 0$$

$\lambda = 1 \quad \lambda = 1 \quad \lambda = 5$ (misma sol que el ejer anterior)

$\boxed{\text{Autovectores}}$

$$\lambda = 1 \longrightarrow A - I_3 = \begin{pmatrix} 1 & 1 & -1 \\ 0 & 1 & -1 \\ -3 & -2 & 2 \end{pmatrix} \longrightarrow \left.\begin{matrix} x + y - z = 0 \\ y - z = 0 \\ \cancel{-3x - 2y + 2z = 0} \end{matrix}\right\} \quad \vec{v} = (0, 1, 1) \longrightarrow \text{No es Diagonalizable}$$

$\hookrightarrow Rg\ A = 2$

No es ya que para un doble valor de $\boxed{\lambda = 1}$, solo encontramos un auto vector independiente.

3. Dada la matriz

$$A = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & -1 & 2 \end{pmatrix}$$

obtenga sus autovalores y autovectores. Razone si es diagonalizable y, en caso afirmativo, escriba las matrices D y P que cumplen que $D = P^{-1}AP$.

$\boxed{\text{Autovalores}}$

$$\det(A - \lambda I_3) = \det\begin{pmatrix} 3-\lambda & 1 & 1 \\ 1 & -\lambda & 1 \\ 0 & -1 & 2-\lambda \end{pmatrix} = \lambda^3 - 5\lambda^2 + 6\lambda$$

	1	-5	6	0
2		2	-6	
	1	-3	0	0

$\boxed{\lambda = 2}$

$\lambda^2 - 3\lambda = 0 \ ; \quad \lambda(\lambda - 3) = 0 \ ; \quad \boxed{\lambda = 0} \quad \boxed{\lambda = 3}$

$\boxed{\text{Autovectores}}$

$$\lambda = 0 \longrightarrow A - 0 = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & -1 & 2 \end{pmatrix} \longrightarrow \left.\begin{matrix} \cancel{3x + y + z = 0} \\ x + z = 0 \\ -y + 2z = 0 \end{matrix}\right\} \quad \vec{v_1} = (-1, 2, 1)$$

$\hookrightarrow Rg\ A = 2$

$$\lambda = 2 \longrightarrow A - 2I_3 = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -2 & 1 \\ 0 & -1 & 0 \end{pmatrix} \longrightarrow \left.\begin{matrix} x + y + z = 0 \\ \cancel{x - 2y + z = 0} \\ -y = 0 \end{matrix}\right\} \quad \vec{v_2} = (-1, 0, 1)$$

$\hookrightarrow Rg\ A = 2$

$$\lambda = 3 \longrightarrow A - 3I_3 = \begin{pmatrix} 0 & 1 & 1 \\ 1 & -3 & 1 \\ 0 & -1 & -1 \end{pmatrix} \longrightarrow \left.\begin{matrix} y + z = 0 \\ x - 3y + z = 0 \\ \cancel{-y - z = 0} \end{matrix}\right\} \quad \vec{v_3} = (-4, -1, 1)$$

$\hookrightarrow Rg\ A = 2$

$\boxed{\text{Diagonalizar}}$ $D = P^{-1} A P$

$0 \cdot (-1, 2, -1) = 0 \cdot (-1, 2, -1) + 0 \cdot (-1, 0, 1) + 0 \cdot (-4, -1, 1)$

$2 \cdot (-1, 0, 1) = 0 \cdot (-1, 2, -1) + 2 \cdot (-1, 0, 1) + 0 \cdot (-4, -1, 1)$

$3 \cdot (-4, -1, 1) = 0 \, (-1, 2, -1) + 0 \cdot (-1, 0, 1) + 3 \cdot (-4, -1, 1)$

$$D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$$P = \begin{pmatrix} -1 & -1 & -4 \\ 2 & 0 & -1 \\ -1 & 1 & 1 \end{pmatrix}$$